USN | | | | | | | | | | 10CS34

Third Semester B.E. Degree Examination, June/July 2014

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. For any three sets A, B, C, prove: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. **(06 Marks)**

b. Among the integers from 1 to 200, find the number of integers that are:
i) not divisible by 5
ii) divisible by 2 or 5 or 9
iii) not divisible by 2 or 5 or 9. **(07 Marks)**

c. A problem is given to four students A, B, C, D whose chances of solving it are 1/2, 1/3, 1/4, 1/5 respectively. Find the probability that the problem is solved. **(07 Marks)**

2 a. Define a tautology and contradiction. Prove that, for any propositions p, q, r, the compound proposition $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology. **(06 Marks)**

b. Define the dual of logical statement. Verify the principle of duality for the following logical equivalence: $[\neg(p \wedge q) \to \neg p \vee (\neg p \vee q)] \Leftrightarrow (\neg p \vee q)$. **(07 Marks)**

c. Define converse, inverse and contra-positive of a conditional with truth table. Write down the contra-positive of $[p \to (q \to r)]$ with:
i) only one occurrence of the connective $\to$
ii) no occurrence of the connective $\to$. **(07 Marks)**

3 a. Negate and simplify each of the following:
i) $\exists x, [p(x) \vee q(x)]$
ii) $\forall x, [p(x) \wedge \neg q(x)]$
iii) $\forall x, [p(x) \to q(x)]$ **(06 Marks)**

b. Establish the validity of the following argument:

$$\begin{array}{l} \forall x, [p(x) \vee q(x)] \\ \underline{\forall x, [\{\neg p(x) \wedge q(x)\} \to r(x)]} \\ \therefore \forall x, [\neg r(x) \to p(x)] \end{array}$$

(07 Marks)

c. Prove that every even integer n with $2 \le n \le 26$ can be written as a sum of atmost three perfect squares. **(07 Marks)**

4 a. Let $a_0 = 1$, $a_1 = 2$, $a_2 = 3$ and $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for $n \ge 3$. Prove that $a_n \le 3^n$ for all positive integers n. **(06 Marks)**

b. Find an explicit definition of the sequence defined recursively by $a_1 = 7$, $a_n = 2a_{n-1} + 1$ for $n \ge 2$. **(07 Marks)**

c. The Lucas numbers are defined recursively by $L_0 = 2$, $L_1 = 1$ and $L_n = L_{n-1} + L_{n-2}$ for $n \ge 2$. Evaluate L_2 to L_{10}. **(07 Marks)**

PART – B

5 a. Suppose A, B, C $\subseteq$ Z X Z with A = {(x, y)|y = 5x – 1}; B = {(x, y)|y = 6x};
C = {(x, y)|3x – y = -7}. Find: (i) $A \cap B$, (ii) $B \cap C$, (iii) $\overline{\overline{A} \cup \overline{C}}$, (iv) $\overline{B} \cup \overline{C}$. **(06 Marks)**

b. Define stirling number of second kind. Find the number of ways of distributing four distinct objects among three identical containers with some containers possibly empty. **(07 Marks)**

c. If f : A →B, g : B → C, and h : C → D are three functions then prove that $(h \circ g) \circ f = h \circ (g \circ f)$. **(07 Marks)**

6 a. Let A = {1, 2, 3, 4}, B = {w. x. y. z} and C = {5, 6, 7}. Also, let R_1 be a relation from A to B, defined by R_1 = {(1, x), (2, x), (3, y), (3, z)} and R_2 and R_3 be relations from B to C, defined by R_2 = {(w, 5), (x, 6)}, R_3 = {(w, 5), (w, 6)}. Find $R_1 \circ R_3$. **(06 Marks)**

b. Find the number of equivalence relations that can be defined on a finite set A with |A| = 6. **(07 Marks)**

c. For A = {a, b, c, d, e}, the Hasse diagram for the poset (A, R) is as shown below:

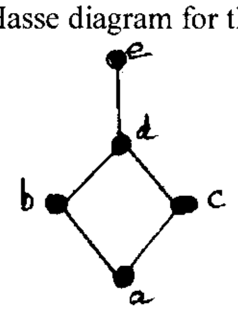

Fig.Q6(c)

i) Determine the relation matrix for R.
ii) Construct the diagraph for R. **(07 Marks)**

7 a. Define subgroup of a group. Let G be a group and let J = { x ∈ G | xy = yx for all y ∈ G}. Prove that J is a subgroup of G. **(06 Marks)**

b. State and prove Lagrange's theorem. **(07 Marks)**

c. The word c = 1010110 is sent through a binary symmetric channel. If p = 0.02 is the probability of incorrect receipt of a signal, find the probability that c is received as r = 1011111. Determine the error pattern. **(07 Marks)**

8 a. The parity-check matrix for an encoding function $E : z_2^3 \rightarrow z_2^6$ is given by

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

i) Determine the associated generator matrix.
ii) Does this code correct all single errors in transmission? **(06 Marks)**

b. Prove that the set z with binary operations ⊕ and ⊙ defined by x ⊕ y = x + y – 1; x ⊙ y = x + y – xy is a cumulative ring. **(07 Marks)**

c. Show that z_6 is not an integral domain. **(07 Marks)**

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